SAINT IGNATIUS' COLLEGE
RIVERVIEW

2012

Higher School Certificate
Trial Examination

Extension 1 Mathematics

General Instructions
Reading time – 5 mins

- Working time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided with this paper
- All necessary working should be shown in every question

Total Marks – 70

Section 1 – 10 marks
Objective response answers
Attempt Questions 1 -10

Section 2 – 60 marks
Extended response answers
Attempt Questions 11 – 14

All questions in Section 2 are of equal value

Section 1 – Objective response answers. (10 marks)

Q1. What is the primitive of cos^2x?

a) $\frac{4x-sin4x}{4}+c$

b) $\frac{1}{2}sin2x+x+c$

c) $\frac{sin2x+2x}{4}+c$

d) $x-\frac{1}{4}sin2x+c$

Q2. The value of the term independent of x in the binomial expansion $\left(x^2+\frac{3}{x}\right)^6$ is

a) 1215

b) 1944

c) 0

d) 2025

Q3. The curves $y=x^2-4$ and $y=x^2-8x+12$ intersect at point Q (2, 0). The acute angle at which the tangents intersect at point Q (2, 0) is

a) $82^0\ 52'$

b) $61^0\ 56'$

c) $7^0\ 8'$

d) $28^0\ 4'$

Q4. What are the coordinates of the point P which divides the interval AB externally in the ratio 5 : 2, where A (3, -1) and B (9, 2)

a) $\mathrm{P}(13, 4)$

b) $\mathrm{P}(1, 4)$

c) $\mathrm{P}\left(\frac{51}{7}, \frac{8}{7}\right)$

d) $\mathrm{P}(13, 3)$

Q5. The solutions to the inequality $\frac{2}{x-5} > 3$ are

a) $x < 5,\ x > \frac{17}{3}$

b) $5 < x < \frac{17}{3}$

c) $x > \frac{17}{3}$

d) $x < \frac{17}{3}$

Q6. The value of $cot\left(sin^{-1}\frac{5}{13}\right)$ is

a) $\frac{5}{12}$

b) $\frac{13}{12}$

c) $\frac{12}{13}$

d) $\frac{12}{5}$

Q7. A root of the Polynomial $P(x) = x^3 - 2x^2 - 5x + 6$ is

a) 2

b) -3

c) 1

d) -4

Q8. The Vertex of the parabola with parameters

$x = p^2 + 2p + 2$ and $y = p - 1$, is

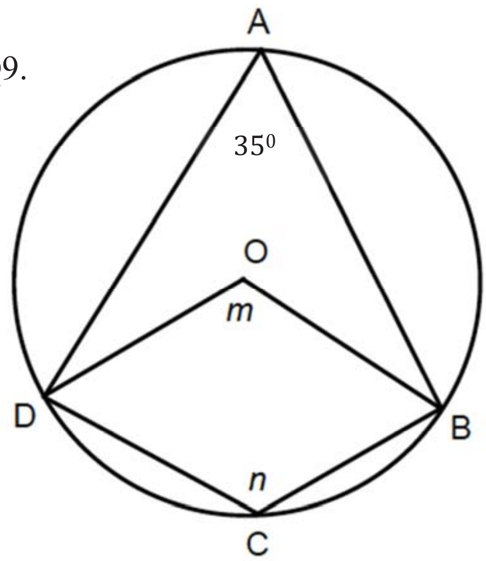
a) $V(0, -1)$

b) $V(2, -1)$

c) $V(1, -2)$

d) $V(1, 0)$

Q9.



The values of the angles labeled as m and n in the diagram are:

a) m = 70, n = 145

b) m = 140, n = 70

c) m = 70, n = 140

d) m = 145, n = 70

Q10. By using the substitution $x = \sin(u)$ or otherwise, the value of $\int_0^{\frac{1}{2}} \sqrt{1-x^2}dx$ is equal to

a) $\frac{\sqrt{3}}{4} + \frac{\pi}{6}$

b) $\frac{\sqrt{3}}{8} + \frac{\pi}{12}$

c) $\frac{\sqrt{3}}{4} + \frac{\pi}{12}$

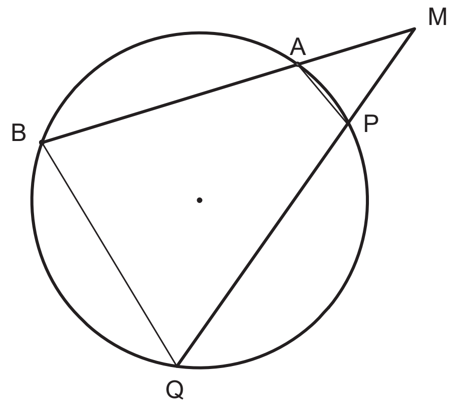
d) $\frac{\sqrt{3}}{8} + \frac{\pi}{6}$

End of Section 1

Section 2 – Extended response answers (60 marks)

Question 11. (Begin a new writing booklet) (15 marks)

a) Let M be a point outside a circle. MAB and MPQ are secants to the circle



i) Prove that $\Delta APM \; ||| \; \Delta QBM$. (2)

ii) Hence, show that $AM \times MB = PM \times MQ$. (1)

b) The tide level in a harbour oscillates according to simple harmonic motion. At 5am, the tide is at its lowest level at 3m and at 11am, the tide rises to its peak at 6m.

i) Calculate the amplitude and the period of motion. (2)

ii) The motion can be written in the form $x - b = a\cos(nt)$ where a, b and n are constants, x represents the tide level in meters and t is the number of hours after 5am. Explain why $a = -1.5, b = 4.5 \; and \; n = \frac{\pi}{6}$. (3)

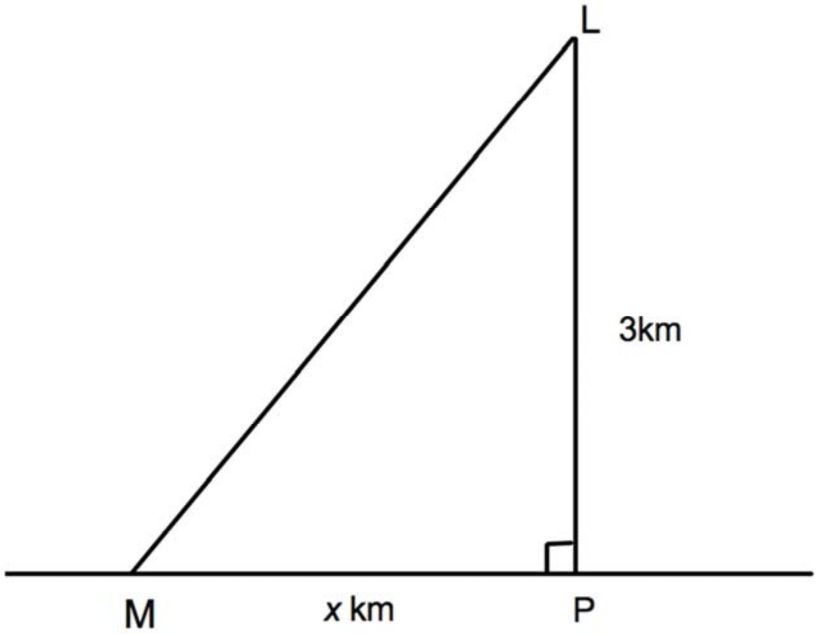
iii) Show that it satisfies the condition $\ddot{x} = -n^2(x - b)$ (1)

iv) What is the first time period (to the nearest minute) during the same day, that a boat can enter and leave the harbour, if the hull of the ship requires a minimum depth of 4m for safe passage? (3)

c) Prove by mathematical induction that $9^{n+2} - 4^n$ is divisible by 5 for $n > 0$ (3)

Question 12. (Begin a new writing booklet) (15 marks)

a) The diagram shows a lighthouse L out at sea, containing a revolving beacon, which is 3km from P, the nearest point on a straight shoreline. The light beam rotates at 4 revs/min and shines a spot of light onto the shoreline at M. M is x km from P and $\angle MLP = \theta$.



i) Why is $\frac{d\theta}{dt} = 8\pi$, where t is the time in minutes. (1)

ii) Write an expression for $\frac{dx}{dt}$. (in terms of θ) (1)

iii) Evaluate $\frac{dx}{dt}$ when the spot of light is at P. (ie $\theta = 0$) (1)

iv) How fast is the spot of light moving 2 km away from P along the shoreline?

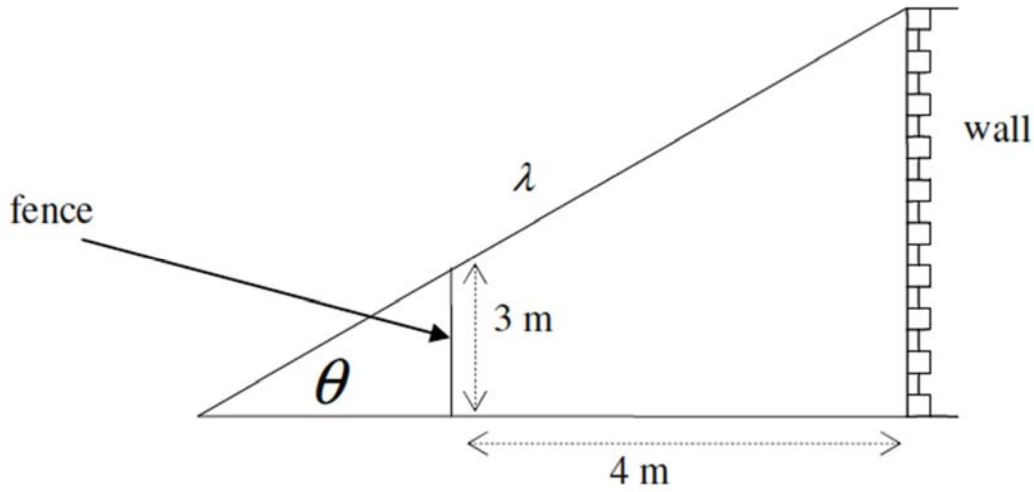
(leave your answer as an exact value) (2)

b) Differentiate $\tan^{-1}(e^{-2x})$ (2)

c) Solve the following equations simultaneously. (2)

$$2\cos^{-1}x + \frac{1}{3}\sin^{-1}y = \frac{\pi}{6}$$
$$\cos^{-1}x - \frac{1}{3}\sin^{-1}y = \frac{\pi}{3}$$

d) A Ladder λ m long is leaning against a vertical wall so that it just touches the top of a fence that is 3m high and 4m from the wall. The ladder is inclined at θ radians to the horizontal.



i) Write the expressions for $sin2\theta$ and $Rsin(\theta + \phi)$ (2)

ii) Prove that the length of the ladder is given by the expression (2)
$\lambda = \frac{3}{sin\theta} + \frac{4}{cos\theta}$

iii) Show that if $\lambda = 10m$ then the angle θ satisfies the equation
$\sin(2\theta) = \sin(\theta + \phi)$ where $\phi = \tan^{-1}\left(\frac{3}{4}\right)$ (2)

Question 13. (Begin a new writing booklet) (15 marks)

(a) The polynomial $P(x) = 4x^3 + 2x^2 + 1$ has one real root in the interval $-1 < x < 0$.

i) Show that there is a root between -0.5 and -1. (1)

ii) Use Newton's method once for x=-0.8 to obtain another approximation to the root. (to 2 decimal places) (2)

(b)The polynomial $P(x) = 2x^3 + 3x^2 - 5x + 7$ has roots. α, β *and* γ.
Evaluate

(2)

i) $\alpha + \beta + \gamma$

ii) $\alpha\beta\gamma$

(b) Consider the function $f(x) = (1 + x)^n$

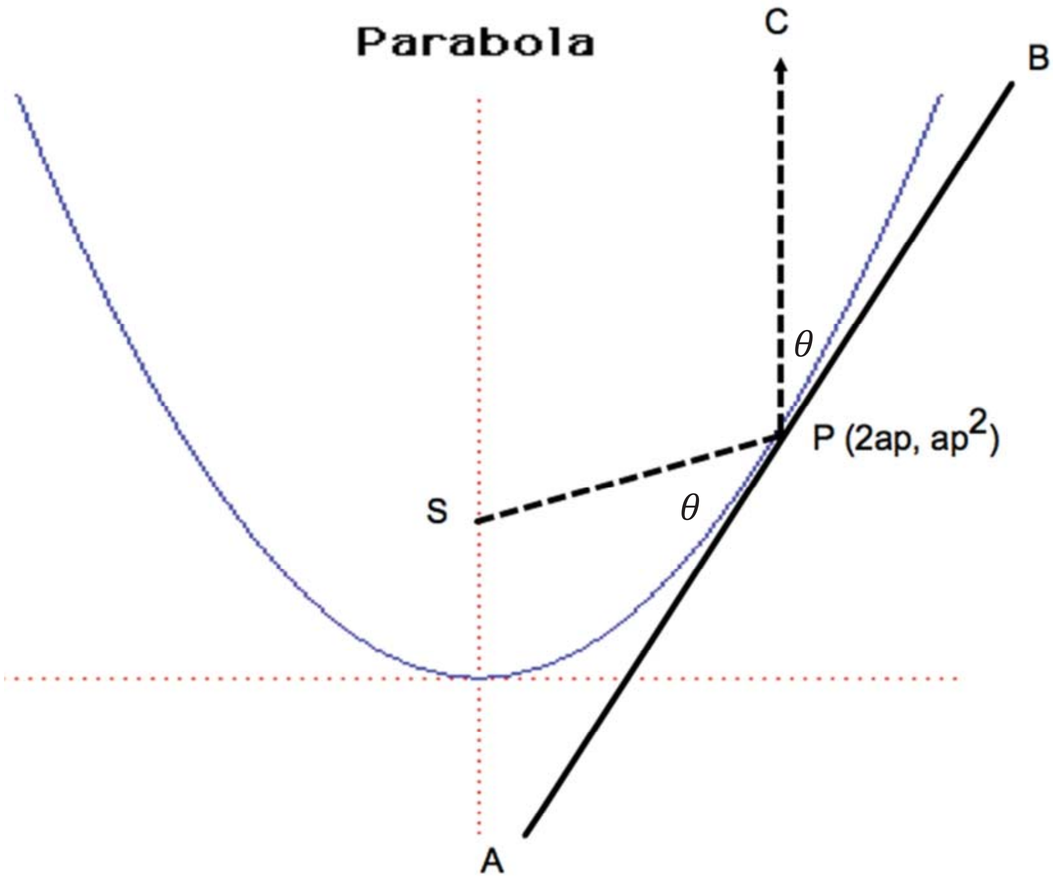
i) Write an expression for $f'(x)$ (1)

ii) By considering the expansion of $(1 + x)^n$ and part (i) above, prove (3)

$$\binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \cdots + n\binom{n}{n-1} + (n+1)\binom{n}{n} = (n+2)2^{n-1}$$

(d) The reflective property of a Parabola suggests that if a ray is constructed from the focus S to any point on the parabola P, then it will be reflected so that it is parallel to the axis of the Parabola.

It also means that if a tangent is drawn at point P, the angle between the tangent and the focus is equal to the angle between the tangent and the reflected ray. (See diagram below)



i) Show that the equation of the tangent AB at point $P\ (2ap, ap^2)$ to the parabola $x^2 = 4ay$ is $px - y - ap^2 = 0$. (2)

ii) By calculating the gradient of PS, show that the expression for the angle between the tangent and the focus S (0, a) at Point P $\angle APS$, is

$$\theta = \tan^{-1}\left(\frac{1}{p}\right)$$ (2)

iii) Prove that the size of $\angle BPC$ is also $\theta = \tan^{-1}\left(\frac{1}{p}\right)$ (2)

Question 14. (Begin a new writing booklet) (15 marks)

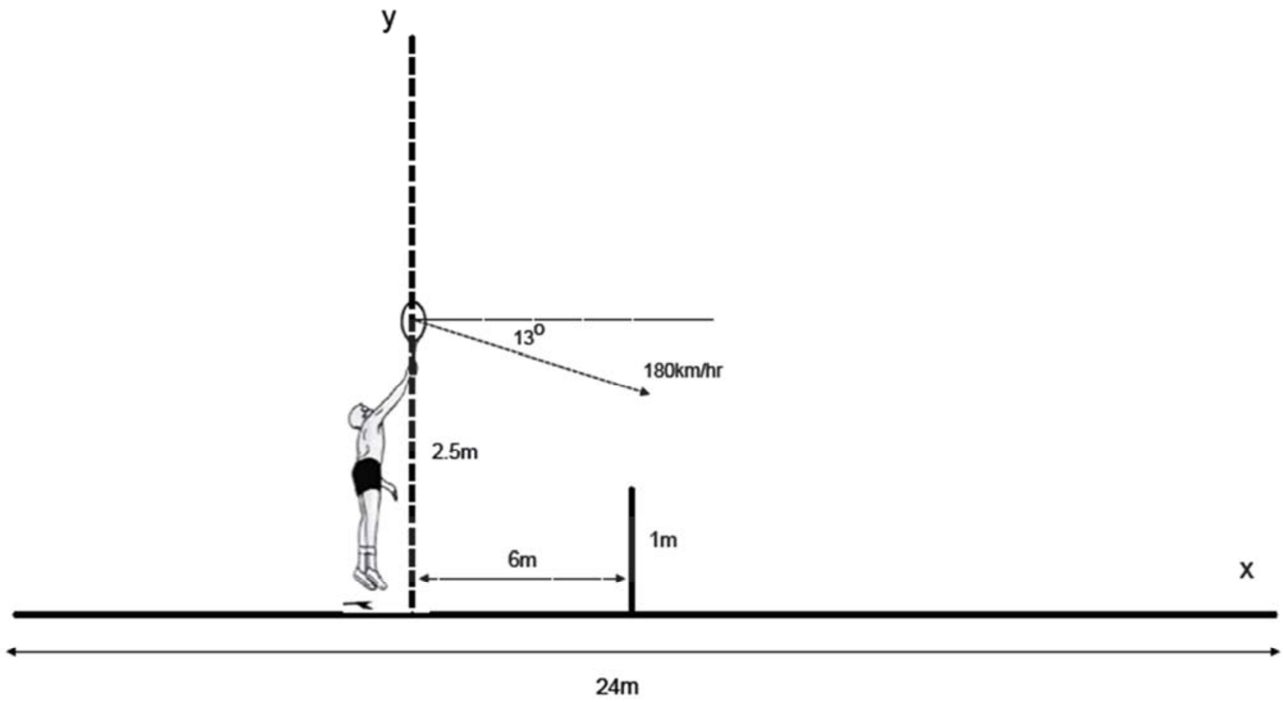
a) Find the general solution of $tan3x = -\sqrt{3}$. (2)

b) Calculate the exact value of $\tan(105^0)$ as a simplified surd. (2)

c) $\int \frac{(x+1)dx}{4x^2+8x-7}$ using the substitution $u = 4x^2 + 8x - 7$ (2)

d) A tennis court is 24m long with a net 1 m high in the middle.
During a tennis match, Rafael Nadal smashes a ball into the opposing court 6 m away from the net at a velocity of 50 m/sec. The ball is projected at an angle of depression of 13^0 at a height of 2.5 m above the ground.

Let g = 10m/s^{-2}. You may also assume that the horizontal displacement of the ball to be given by the expression $x = 50tcos13^0$, where *t* is the time in sec and *x* is in metres.

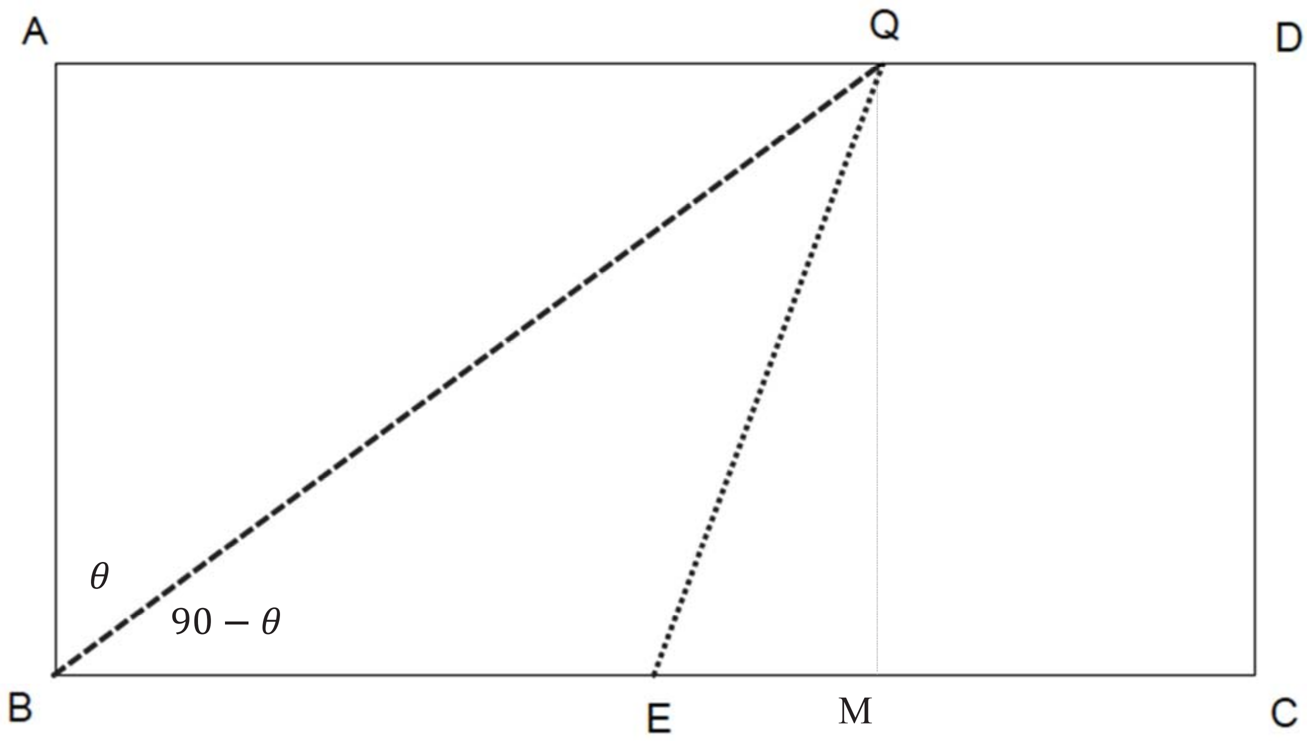


i) Given that $\ddot{y} = -g$, show that the vertical displacement of the ball at time *t* is $y = -5t^2 - 50tsin13^0 + 2.5$ (1)

ii) By what margin does the ball clear the net? (2)
(correct to the nearest cm)

iii) How far from the opposing player's baseline does the ball land (3)
(correct to the nearest cm)

e) A rectangular field ABCD has the dimensions 60m by 30 m where AB = CD = 30m. E is the midpoint of BC.
At the same time, one person leaves point B cycling at 3m/s and another leaves from point E jogging at 2m/s.
They travel towards the other side of the field to arrive simultaneously at point Q.
Let $\angle ABQ = \theta$ and let the distance $QE = x$
Diagram is given below.



i) By equating equal travel times, show that $QB = \frac{3x}{2}$ (1)

ii) M lies on EC and is the foot of the perpendicular from Q. By considering ΔQAB and ΔQBE, prove the following (2)

$$5x^2 + 3600 = 360\sqrt{x^2 - 400}$$

End of paper

Student's Name: ___________________________ Teacher's Name: _____________________________

Year 12 Mathematics Ext 1 – Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question.
Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ● B ~~●~~ C ○ D ○

---

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

SAINT IGNATIUS' COLLEGE
RIVERVIEW

Suggested Solutions

2012
Higher School Certificate
Trial Examination

Extension 1 Mathematics.

General Instructions
Reading time – 5 mins

- Working time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided with this paper
- All necessary working should be shown in every question

Total Marks – 70

Section 1 – 10 marks
Objective response answers
Attempt Questions 1 -10

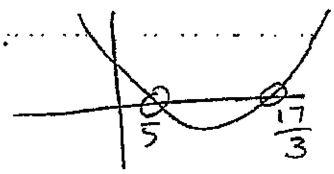
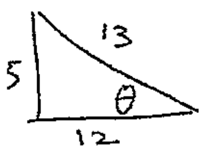
Section 2 – 60 marks
Extended response answers
Attempt Questions 11 – 14

All questions in Section 2 are of equal value

Mathematics Extension 1: Question Objective Response (Section 1)

Suggested Solutions	Marks Awarded	Markers Comments
1) $\int \cos^2 x$ $= \int \frac{1}{2}(\cos 2x + 1)\,dx$ $= \frac{1}{4}\sin 2x + \frac{x}{2} + c$ $= \frac{\sin 2x + 2x}{4} + c$ C	1	
2) $T_K = {}^6C_{K-1}(x^2)^{6-K+1}(3x^{-1})^{K-1}$ $= {}^6C_{K-1}(x^2)^{7-K}(3)^{K-1}(x)^{1-K}$ $= {}^6C_{K-1}\,3^{K-1}\,x^{14-2K}\,x^{1-K}$ $= {}^6C_{K-1}\,3^{K-1}\,x^{15-3K}$ $\therefore K = 5$ $\therefore T_5 = {}^6C_4\,3^4$ $= (15)(81)$ $= 1215$ A	1	
3) $y = x^2 - 4$ $y' = 2x$ at $x = 2$ $y' = 4$ $y = x^2 - 8x + 12$ $y' = 2x - 8$ at $x = 2$ $y' = 4 - 8$ $y' = -4$ $\therefore \tan\theta = \left\lvert \frac{4 - -4}{1 + (4)(-4)} \right\rvert$ $= \left\lvert \frac{8}{-15} \right\rvert$ $\therefore \theta = 28^\circ 4'$ D	1	

Mathematics Extension 1: Question Objective Response (Section 1)

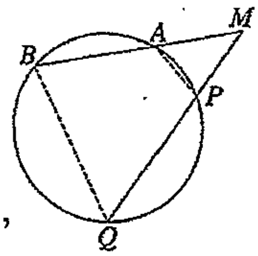
Suggested Solutions	Marks Awarded	Markers Comments
4 $P(a,b) = \left(\dfrac{5(9) - 2(3)}{5-2} , \dfrac{5(2) - 2(-1)}{5-2} \right)$	1	
$= \left(\dfrac{39}{3} , \dfrac{12}{3} \right)$		
$= (13, 4)$ A		
5 $\dfrac{2}{x-5} > 3 \qquad \times (x-5)^2$		
$2(x-5) > 3(x-5)^2$	1	
$0 > 3(x-5)^2 - 2(x-5)$		
$0 > (x-5)[3(x-5) - 2]$		
$0 > (x-5)(3x-17)$		
∴  B		
$\therefore 5 < x < \frac{17}{3}$		
6  $\therefore \cot\left(\sin^{-1}\frac{5}{13}\right) = \frac{12}{5}$ D	1	

Suggested Solutions	Marks Awarded	Markers Comments
(7) $P(x) = x^3 - 2x^2 - 5x + 6$ $P(1) = 1 - 2 - 5 + 6$ $= 0$ $\therefore x = 1$ is a root. (C)	1	
(8) $x - 1 = p^2 + 2p + 1$ $(x-1) = (p+1)^2$ also $y + 1 = p$ $\therefore (x-1) = (y+1+1)^2$ $(x-1) = (y+2)^2$ $\therefore V: (1, -2)$ (C)	1	
(9) $m = 70^\circ$ ($\angle$ at centre, $\times 2$ $\angle$ at circumference) $n = 145^\circ$ (Opp $\angle$'s in cyclic are supplementary) (A)	1	
(10) $\int_0^{\frac{1}{2}} \sqrt{1-x^2}\, dx$ $\quad x = \sin u$, $dx = \cos u\, du$ $x = 0 \quad u = 0$. $x = \frac{1}{2} \quad u = \frac{\pi}{6}$ $\therefore \int_0^{\frac{\pi}{6}} \sqrt{1 - \sin^2 u}\, \cos u\, du$ $= \int_0^{\frac{\pi}{6}} \cos^2 u\, du$ $= \frac{1}{2}\left[\frac{\sin 2u}{2} + u\right]_0^{\frac{\pi}{6}}$ $= \frac{\sqrt{3}}{8} + \frac{\pi}{12}$ (B)	1	

Mathematics Extension 1: Question 11

Suggested Solutions	Marks Awarded	Markers Comments

GIVEN: Let M be a point outside a circle, and let MAB and MPQ be secants to the circle.



AIM: To prove that $AM \times MB = PM \times MQ$.

CONSTRUCTION: Join AP and BQ.

PROOF: In the triangles APM and QBM:

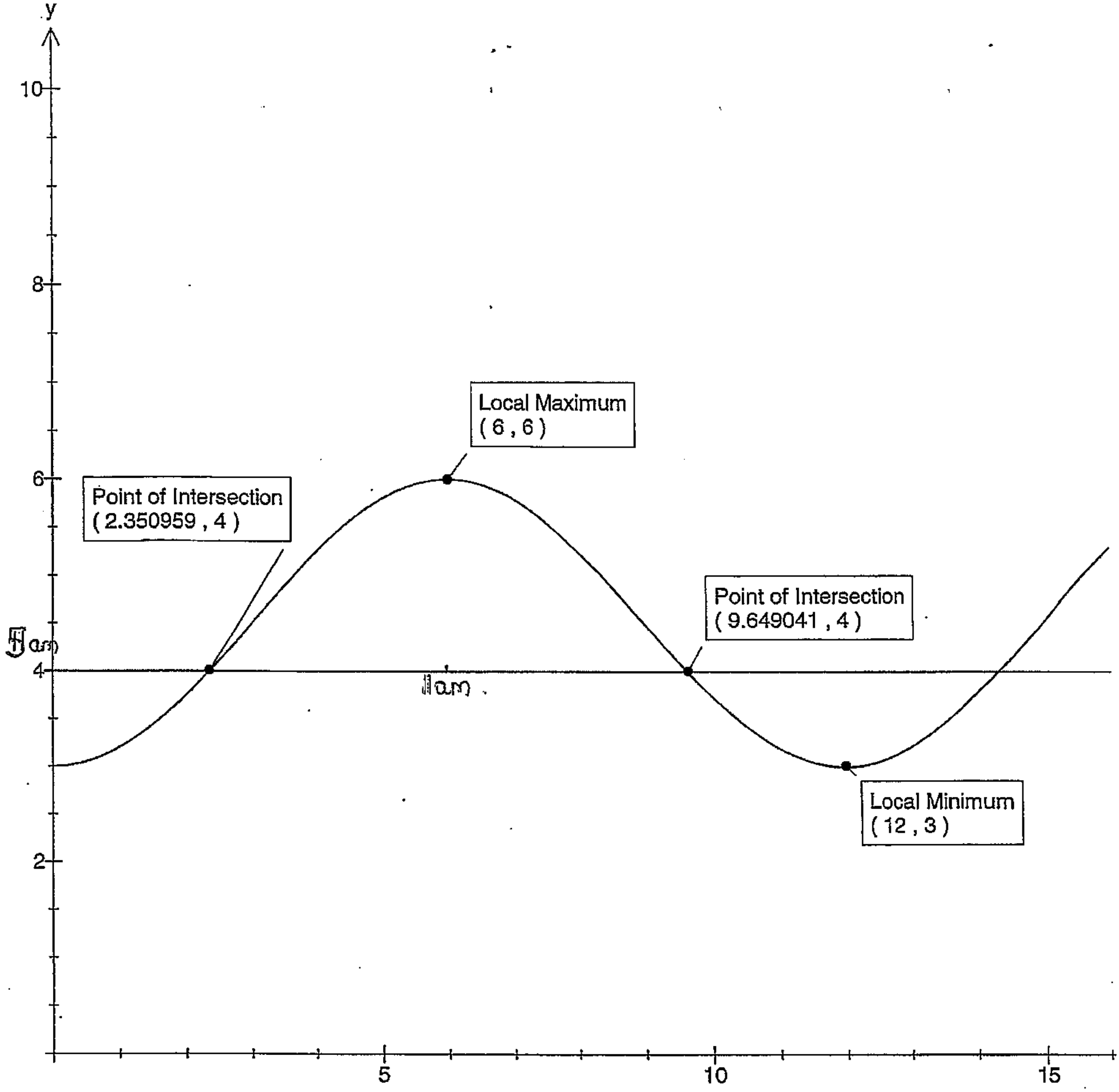
1. $\angle MAP = \angle MQB$ (external angle of cyclic quadrilateral),
2. $\angle AMP = \angle QMB$ (common),

so $\triangle APM \ ||| \ \triangle QBM$ (AA).

Hence $\dfrac{AM}{QM} = \dfrac{PM}{BM}$ (matching sides of similar triangles),

that is, $AM \times MB = PM \times MQ$.

b)		
i) Amplitude $\rightarrow$ (highest − Lowest) $\div 2$ $\therefore (6-3) \div 2 = 1.5\text{m}$	1	
Period = (time between high tide and low tide) $\times 2$ $= (11\text{am} - 5\text{am}) \times 2$ $= 12$ hrs.	1	
ii) $a = -1.5$ (since at $t=0$ tide is at lowest point)	1	
$b = 4.5$ (centre of motion is at 4.5m)	1	
$T = \dfrac{2\pi}{n}$ $12 = \dfrac{2\pi}{n}$ $\therefore n = \dfrac{\pi}{6}$	1	


y
10
8
6
4
2
5
10
15
Local Maximum
(6 , 6)
Point of Intersection
(2.350959 , 4)
Point of Intersection
(9.649041 , 4)
Local Minimum
(12 , 3)
3am
11am

Mathematics Extension 1: Question 11

Suggested Solutions	Marks Awarded	Markers Comments
b) iii) $x - 4.5 = -1.5\cos\left(\frac{\pi}{6}t\right)$		
$\dot{x} = 1.5\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}t\right)$		
$\ddot{x} = 1.5\left(\frac{\pi}{6}\right)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}t\right)$	1	
$= -\left(\frac{\pi}{6}\right)^2\left(-1.5\cos\left(\frac{\pi}{6}t\right)\right)$		
$\ddot{x} = -n^2 X$ where $X = (x-4.5)$		
iv) $-1.5\cos\left(\frac{\pi}{6}t\right) + 4.5 > 4$		
$\therefore\ 0.5 > 1.5\cos\left(\frac{\pi}{6}t\right)$		
$\frac{1}{3} > \cos\left(\frac{\pi}{6}t\right)$		
$\therefore \frac{\pi}{6}t = 1.23,\ 2\pi - 1.23,\ 2\pi + 1.23,\ 4\pi - 1.23$ etc.	1	
$\therefore t = 2.35,\ 9.65,\ 14.35,\ 21.65$		
= 2hrs 21min, 9hrs 39min, 14hrs 21min, 21hrs 39min etc.		
$\therefore$ First time period after 5am for safe passage	1	
7.21am → 2.39pm		
c) Prove true for $n = 1$		
$9^{1+2} - 4^1$		
$= 9^3 - 4$	1	
$= 729 - 4$		
$= 725$		
$= (145)(5)$		
$\therefore$ divisible by 5		

Suggested Solutions	Marks Awarded	Markers Comments
c) Assume true for $n=k$		
$9^{k+2} - 4^k = 5A \rightarrow 9^{k+2} = 5A + 4^k$	1	
Prove true for $n=k+1$		
$9^{k+3} - 4^{k+1} = 5B$		
$LHS = 9(9^{k+2}) - 4(4^k)$	1	
$= 9\left[5A + 4^k\right] - 4(4^k)$		
$= 45A + 9(4^k) - 4(4^k)$		
$= 45A + 5(4^k)$		
$= 5\left[9A + 4^k\right]$	1	
$= 5B$ where $B = (9A + 4^k)$		
12 a)		
i) since, 4 revs/min		
$\therefore$ 8π rad/min as 1 rev = 2π rad.	1	
$\therefore \frac{d\theta}{dt} = 8\pi$		
ii) $\tan\theta = \frac{x}{3}$ $\therefore \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$		
$x = 3\tan\theta$ $= 3\sec^2\theta \times 8\pi$	1	
$\frac{dx}{d\theta} = 3\sec^2\theta$. $= 24\pi\sec^2\theta$		
iii) at P, $\theta = 0$		
$\therefore \frac{dx}{dt} = 24\pi\sec^2(0)$	1	
$= 24\pi$ Km/min.		

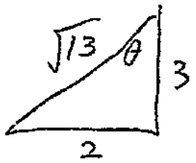
Mathematics Extension 1: Question 12

Suggested Solutions	Marks Awarded	Markers Comments

a)

iv) when $x = 2$

$\tan\theta = \frac{2}{3}$



$\therefore \frac{dx}{dt} = 24\pi \times \left(\frac{\sqrt{13}}{3}\right)^2$ (1)

$= \frac{104\pi}{3}$ km/min (1)

since $\sec\theta = \frac{\sqrt{13}}{3}$

b) $\frac{d}{dx} \tan^{-1}(e^{-2x})$

$= \frac{-2e^{-2x}}{1 + (e^{-2x})^2}$ (1)

$= \frac{-2e^{-2x}}{1 + e^{-4x}} \times \frac{e^{4x}}{e^{4x}}$

$= \frac{-2e^{4x}}{e^{4x} + 1}$ (1)

c) let $a = \cos^{-1}x$ and $b = \sin^{-1}y$

$\therefore 2a + \frac{b}{3} = \frac{\pi}{6}$ (1)

$a - \frac{b}{3} = \frac{\pi}{3}$ (2)

(1) + (2)

$3a = \frac{\pi}{2}$

$a = \frac{\pi}{6}$

$\therefore \cos^{-1}x = \frac{\pi}{6}$

$x = \frac{\sqrt{3}}{2}$

$\therefore 2\left(\frac{\pi}{6}\right) + \frac{b}{3} = \frac{\pi}{6}$

$\frac{b}{3} = -\frac{\pi}{6}$

$b = -\frac{\pi}{2}$

$\sin^{-1}y = -\frac{\pi}{2}$

$y = -1$

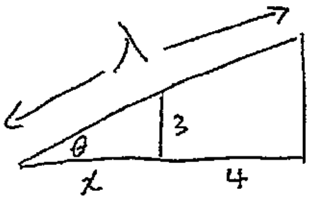
(2) for both solutions.

Mathematics Extension 1: Question 12

Suggested Solutions	Marks Awarded	Markers Comments

d)

i)



$\tan\theta = \dfrac{3}{x}$

$\therefore x = \dfrac{3}{\tan\theta}$ 1

$\therefore \cos\theta = \dfrac{x+4}{\lambda}$

$\therefore \lambda = \dfrac{x}{\cos\theta} + \dfrac{4}{\cos\theta}$

$\lambda = \dfrac{\frac{3}{\tan\theta}}{\cos\theta} + \dfrac{4}{\cos\theta}$ 1

$\lambda = \dfrac{3}{\sin\theta} + \dfrac{4}{\cos\theta}$ as required

ii)

$\sin 2\theta = 2\sin\theta\cos\theta$ 1

$R\sin(\theta+\phi) = R[\sin\theta\cos\phi + \cos\theta\sin\phi]$ 1

iii) let $\lambda = 10$

$\therefore 10 = \dfrac{3}{\sin\theta} + \dfrac{4}{\cos\theta} \quad \times \sin\theta\cos\theta$

$10\sin\theta\cos\theta = 3\cos\theta + 4\sin\theta$ 1

$5[2\sin\theta\cos\theta] = 5[\sin(\theta+\phi)]$ 1

$\therefore$

$\underline{\sin 2\theta = \sin(\theta+\phi)}$

since in $4\sin\theta + 3\cos\theta$

$R = \sqrt{4^2+3^2}$

$= 5$

and $\phi = \tan^{-1}\left(\frac{b}{a}\right)$

$\phi = \tan^{-1}\left(\frac{3}{4}\right)$

Suggested Solutions	Marks Awarded	Markers Comments

a) i) $P(x) = 4x^3 + 2x^2 + 1$

$P'(x) = 12x^2 + 4x$

$\therefore P(-0.5) = 4(-0.5)^3 + 2(-0.5)^2 + 1$

$= -1$

$P(-1) = 4(-1)^3 + 2(-1)^2 + 1$

$= 1$

$\therefore$ a root lies between $x = -0.5$ and $x = 1$ — (1)

ii) $x_2 = -0.8 - \dfrac{P(-0.8)}{P'(-0.8)}$ — (1)

$= -0.8 - \dfrac{0.232}{12(-0.8)^2 + 4(-0.8)}$

$= -0.85$ (2dp) — (1)

b) i) $\alpha + \beta + \gamma = -\dfrac{b}{a}$

$= -\dfrac{3}{2}$ — (1)

ii) $\alpha\beta\gamma = -\dfrac{d}{a}$

$= -\dfrac{7}{2}$ — (1)

c) $f(x) = (1+x)^n$

i) $f'(x) = n(1+x)^{n-1}$ — (1)

ii)

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_{n-1} x^{n-1} + {}^nC_n x^n$

Derive

$n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2 x + 3\,{}^nC_3 x^2 + \dots + n\,{}^nC_n x^{n-1}$ — (1)

Suggested Solutions	Marks Awarded	Markers Comments
add above Expressions. and let $x=1$		
$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots {}^nC_{n-1} + {}^nC_n$	1	
$n(2)^{n-1} = {}^nC_1 + 2\,{}^nC_2 + 3\,{}^nC_3 + \dots + (n-1)\,{}^nC_{n-1} + n\,{}^nC_n$		
$2^n + n(2^{n-1}) = {}^nC_0 + 2\,{}^nC_1 + 3\,{}^nC_2 + \dots + n\,{}^nC_{n-1} + (n+1)\,{}^nC_n$		
$2^{n-1}[2+n] = \sum_{k=0}^{n} (k+1)\,{}^nC_k$ as required.	1	
d) i) $x = 2ap$, $y = ap^2$		
$\frac{dx}{dp} = 2a$ $\quad \frac{dy}{dp} = 2ap$		
$\therefore \frac{dy}{dx} = \frac{dy}{dp} \times \frac{dp}{dx}$	1	
$= 2ap \times \frac{1}{2a}$		
$= p$		
$\therefore p = \frac{y - ap^2}{x - 2ap}$		
$xp - 2ap^2 = y - ap^2$	1	
$\therefore px - y - ap^2 = 0$ as required.		
ii) $m_{PS} = \frac{ap^2 - a}{2ap - 0}$		
$= \frac{a(p^2-1)}{2ap}$	1	
$m_{PS} = \frac{p^2-1}{2p}$		

Suggested Solutions	Marks Awarded	Markers Comments
d) ii) Angle between 2 lines.		
$\tan\theta = \left\| \dfrac{p - \frac{p^2-1}{2p}}{1 + p\left(\frac{p^2-1}{2p}\right)} \right\| \begin{matrix} \times 2p \\ \times 2p \end{matrix}$		
$= \left\| \dfrac{2p^2 - p^2 + 1}{2p + p^3 - p} \right\|$	1	
$= \left\| \dfrac{(p^2+1)}{p(p^2+1)} \right\|$		
$\tan\theta = \dfrac{1}{p}$		
iii) Construct a horizontal from P away from axis of Parabola to D		
$\therefore \angle BPD = 90 - \theta$ and $m_{PD} = 0$	1	
$\therefore \tan(90-\theta) = \left\| \dfrac{p - 0}{1 + p(0)} \right\|$		
$= p$		
But $\tan(90-\theta) = \cot\theta$		
$\therefore \cot\theta = p$	1	
$\dfrac{1}{\tan\theta} = p$		
$\therefore \tan\theta = \dfrac{1}{p}$ as required		

Mathematics Extension 1: Question 14

Suggested Solutions	Marks Awarded	Markers Comments
a) $\tan 3x = -\sqrt{3}$ $\therefore$ 2nd, 4th Quad. $180-\theta$, $360-\theta$		
$\therefore 3x = 120, 300, 480, 660 \ldots\ldots$	1	
$= \pi - \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, 3\pi - \frac{\pi}{3}, 4\pi - \frac{\pi}{3} \ldots$		
$\therefore x = \frac{\pi}{3} - \frac{\pi}{9}, \frac{2\pi}{3} - \frac{\pi}{9}, \frac{3\pi}{3} - \frac{\pi}{9}, \frac{4\pi}{3} - \frac{\pi}{9} \ldots$		
$= \frac{n\pi}{3} - \frac{\pi}{9}$ for all integers $n \geq 1$	1	
b) $\tan(105) = \tan(60+45)$		
$= \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45}$	1	
$= \frac{\sqrt{3}+1}{1-(\sqrt{3})(1)} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$	1	
$= \frac{3+2\sqrt{3}+1}{1-3}$		
$= \frac{4+2\sqrt{3}}{-2} = -2-\sqrt{3}$	1	
c) $\int \frac{(x+1)\,dx}{4x^2+8x-7}$		
let $u = 4x^2 + 8x - 7$		
$du = 8x + 8\ dx$		
$du = 8(x+1)\,dx$	1	
$\therefore \frac{1}{8} \int \frac{du}{u}$		
$= \frac{1}{8} \ln [u] + c$		
$= \frac{1}{8} \ln [4x^2 + 8x - 7] + C$	1	

Suggested Solutions	Marks Awarded	Markers Comments
d) $\ddot{y} = -10$		
i) $\dot{y} = -10t + c$		
when $t=0$ $\dot{y} = -50\sin 13$		
$\therefore c = -50\sin 13$		
$\therefore \dot{y} = -10t - 50\sin 13$		
$y = -5t^2 - 50t\sin 13 + c$	1	
when $t=0$ $y = 2.5$		
$\therefore c = 2.5$		
$\therefore y = -5t^2 - 50t\sin 13 + 2.5$		
ii) since $x = 50t\cos 13$		
$t = \dfrac{x}{50\cos 13}$		
at $x=6$	1	
$t = \dfrac{6}{50\cos 13}$		
$= 0.1232$ sec.		
$\therefore y = -5(0.1232)^2 - 50(0.1232)\sin 13 + 2.5$		
$= 1.039$ m.	1	
$\therefore$ clear net by 0.039m $\doteqdot 3.9$cm.		
iii) let $y=0$		
$\therefore 5t^2 + 50t\sin 13 - 2.5 = 0$		
$t = \dfrac{-50\sin 13 \pm \sqrt{(50\sin 13)^2 - 4(5)(-2.5)}}{10}$		
$= 0.2038,\ -2.45$ ← invalid time.	1	
$\therefore x = 50(0.2038)\cos 13$		
$= 9.93$ m.		

Suggested Solutions	Marks Awarded	Markers Comments

d(ii) ∴ Ball lands

$12 - 3.93\text{ m}$

$= 8.07\text{ m}$ from opponents base line — (1)

e) i) Jogger

$T = \frac{D}{S}$

$= \frac{x}{2}$

Cyclist

$D = S \times T$

$= 3 \times \frac{x}{2}$

$= \frac{3x}{2}$ — (1)

ii) In ΔQAB

$(AQ)^2 = \left(\frac{3x}{2}\right)^2 - 30^2$

$= \frac{9x^2}{4} - 900$

$\therefore AQ = \sqrt{\left(\frac{9x^2}{4}\right) - 900}$

$= \frac{3}{2}\sqrt{x^2 - 400}$

$\therefore \sin\theta = \frac{\frac{3}{2}\sqrt{x^2-400}}{\frac{3x}{2}}$

$\sin\theta = \frac{\sqrt{x^2-400}}{x}$ — (1)

$\therefore$ In ΔQBE

$x^2 = \left(\frac{3x}{2}\right)^2 + 30^2 - 2\left(\frac{3x}{2}\right)(30)\cos(90-\theta)$

$= \frac{9x^2}{4} + 900 - 90x\sin\theta$

$= \frac{9x^2}{4} + 900 - 90x\frac{\sqrt{x^2-400}}{x}$

$x^2 = \frac{9x^2}{4} + 900 - 90\sqrt{x^2-400}$ $\quad (\times 4)$ — (1)

$4x^2 = 9x^2 + 3600 - 360\sqrt{x^2-400}$

$\therefore 5x^2 + 3600 = 360\sqrt{x^2-400}$